A better aspheric equation

There is growing momentum in the optical engineering community to change the way we describe aspheric surfaces. The new proposal by Greg Forbes[12] allows us to make aspheric optics which are as powerful as the current general even aspheres for image correction, but which are simpler to optimize, easier to understand, cheaper to manufacture, and easier to test. And while the optics community is not famous for rapid adoption of great new ideas, the speed at which the "Forbes Asphere" is moving is worthy of our attention.

Our equation for general aspheric optics has been around for a while. Karl Schwarzschild published a series of papers in 1905, which ushered in the "conic constant" and a family of aspheric optics for astronomy and microscopy. But the more general equation for aspheric optics dates back 1936, when scientists were trying to understand what Bernhard Schmidt did to correct spherical aberration in his "Schmidtspiegel", or Schmidt camera. Since the corrector was made using a partial vacuum, they added a simple fourth order boilerplate deviation to Schwarzschild's equation, which describes such a deformation reasonably well. The result is the standard equation for general, even aspheric surfaces given as:

$$z = \frac{ch^2}{\left(1+\sqrt{1-(1+k)c^2h^2}\right)} + h^4 \sum_{m=0}^{M} a_m h^{2m} \qquad (1)$$

The problem is that the simple polynomial solution is not very intuitive and is awkward to optimize; since each of the terms are almost identical (figure 1) a reasonable approximation of the surface requires many coefficients of alternating sign. The result is a string of numbers most people do not understand, which cannot be toleranced, and simply must be translated from the computer of the designer to the computers of the manufacturing tools and measurement equipment, hopefully without a typographical error.

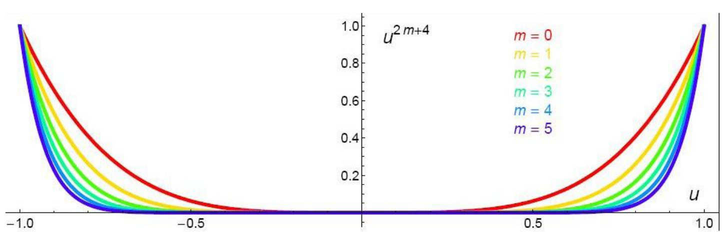

Figure 1. First six terms of the aspheric polynomial in equation 1.

The proposal by Greg Forbes in 2007 was to replace the power series of this equation with a different set of basis functions which are normalized and orthagonalized by combining these different terms in meaningful ways. I won't go into the rationale or derivation here; you can read the papers yourself. But the equation on the drawing described above is simply replaced by:

$$z = \frac{ch^2}{\left(1+\sqrt{1-(1+k)c^2h^2}\right)} + u^4 \sum_{m=0}^{M} a_m Q_m^{con}(u^2) \qquad (2)$$

where u is just h/h_{max} and Q_m^{con} is a series of new basis members, plotted in figure 2.

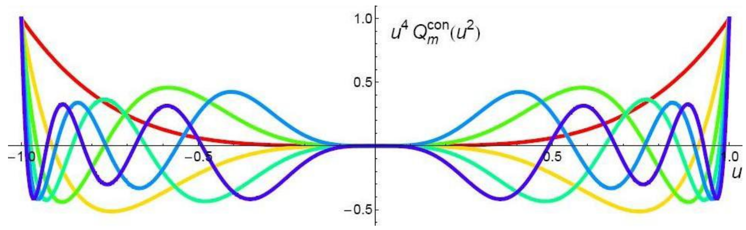

Figure 2. first six terms of aspheric polynomial in equation 2.

Now the coefficients are normalized and the magnitude of the contribution to the surface form is directly reflected in the coefficient itself. Small coefficients mean less departure. Since the basis terms integrate to zero, the optimal value of each of the coefficients is independent of M; adding more terms will not change the optimum coefficients of the lower order terms.

The elegance of this formalism cannot be over-stated. The formulas for the equation above (and another one based on best-fit sphere normalization) have already been incorporated into Zemax® and CodeV® and are consistent with the notation for specifying aspheric optics using ISO 10110-12. All the tools are in place to phase out the simple boiler-plate deformation equation of 1936 and replace it with a new formalism which represents a significantly better aspheric equation.

---

[1] G.W. Forbes, "Shape specification for axially symmetric optical surfaces," Opt. Express **15(8)**, 5218–5226 (2007).

[2] G.W. Forbes and C.P. Brophy, "Asphere, O Asphere, how shall I describe thee?", ProcSPIE **7100**, 710002-1-15 (2008)